

Age of the Universe and the Density Parameter

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An exact formula for the age of the universe in terms of the density parameter is found that may be used instead of the approximate one in current use, among other purposes, for the estimate of an upper bound of relic particles in the present universe.

Dolgov *et al.* (1990) remarked that the energy density of relic particles in the present universe has an upper bound given by the age of the universe, because otherwise this age would be too small. The relation used by these authors is

$$t_u \approx \left[H \left(1 + \frac{r^{1/2}}{2} \right) \right]^{-1} \quad (1)$$

where t_u stands for the age of the present universe, H is Hubble's parameter, and $r = \rho_{\text{tot}}/\rho_{\text{crit}} > \rho_a/\rho_{\text{crit}}$ is the density parameter, where ρ_a is the density of stable particles.

We wish to point out a more exact formula that must be used instead of (1). With this purpose in mind, we point out that the age of the universe may be calculated from the knowledge of how the "radius" of the universe varies with time. Berman (1983) and Berman and Gomide (1988) showed that for a power law of time, say,

$$R \propto t^{1/m} \quad (2)$$

where m is a constant, one finds that

$$m = 1 + q \quad (3)$$

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where q is the deceleration parameter, which is constant. The present value of m is supposed to be nearly

$$m \approx \frac{3}{2} \quad (4)$$

but this does not matter really for us in calculating the desired formula.

For constant deceleration parameter, we have, from Berman (1983) and Berman and Gomide (1988), the following relation:

$$H = [(q + 1)t]^{-1} \quad (5)$$

On the other hand, in the simplest case, Weinberg (1972) and others have calculated

$$r = 2q \quad (6)$$

On combining both relations, one gets

$$t = \left[H \left(1 + \frac{r}{2} \right) \right]^{-1} \quad (7)$$

This is the formula we were looking for, which should be used instead of (1).

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